the cores are pressed on to the sample. The results presented in Table II were obtained using small axis pressure $P=600$ N. With an increase of pressure in the range of 600–6000 N, the gap d_s decreased to about half its size.

2) An interesting dependence was obtained in investigating the imaginary part of permittivity of metalized and nonmetalized samples as a function of pressure. As an example of these measurements, one sample (of ceramic N-47) is presented in Fig. 7.

On the basis of the ϵ'' measurement as a function of pressure, it can be established that a more accurate measure of material losses can be obtained for nonmetalized samples due to the elimination of a significant portion of the contact resistivity (core-metalized layer) in the dissipation of energy. In general, with a metalized layer the ϵ'' results are overestimated; without it they are underestimated. In order to increase accuracy, it is necessary to use significant pressure P, which, however, must be smaller than the force which can destroy the sample. Associated with this is the need to use a hard material for the contact surface of the core, so that it will not be damaged by the hard ceramic sample.

III. Summary

In Part A, a theoretical analysis of the measurement method was present while in Part B, an experimental analysis of measurement error showed that the author's proposed measurement method for thin samples in a reentrant cavity allows for $\hat{\epsilon}$ measurements in a wide range of materials ($\epsilon'=2$–300, $\tan\delta=10^{-5}$–10^{-1}) with an adequate technical accuracy $\delta\epsilon'\leqslant 1$ percent, $\delta(\tan\delta)\leqslant 5$ percent$+5\times10^{-5}$.

References

[1] A. Kaczkowski and A. Milewski, "A high-accuracy wide-range measurement method for determination of complex permittivity n reentrant cavity—Part A: Theoretical analysis of the method," this ssue, pp. 225–228.
[2] A. Milewski, "Q factor measurement of microwave resonators, using binary klystron modulation," (in Polish) *Przegl. Elektron.* no. 10, pp. 498–506, Oct. 1969.

Wave Propagation through Weakly Anisotropic Straight and Curved Rectangular Dielectric Optical Guides

B. B. CHAUDHURI, STUDENT MEMBER, IEEE

***Abstract*—Wave propagation through weakly anisotropic straight and curved dielectric rectangular guides is studied using a coupled mode approach. The propagation constant, thus found, can be computed very easily if Marcatili's approximate field expressions for an isotropic guide are used. The result for the uniaxial case can then be extended for the biaxial crystal to the first order of approximation.**

I. Introduction

Dielectric waveguides of different shapes have found wide use in integrated optics and optical communication systems. The basic structures that have been studied extensively for optical applications are, in general, isotropic slabs, rectangular and circular cylindrical guides. However, in many diverse applications, such as dielectric cavity resonators (DCR), laser and masers, ESR spectroscopy, nonlinear optical devices, etc., the interaction may involve an anisotropic dielectric medium. Symmetrical [1], [2] as well as hybrid [3], [4] mode propagation in a uniaxial anisotropic rod have been extensively studied. The theory has since been extended to the weakly anisotropic case [5], biaxial rod [6], and hollow axially anisotropic structure [7] with possible applications in retinal receptor modeling, crystal-core guides, and gas laser resonating structures, respectively.

The work on planner structures, however, seems incomplete. The effect of anisotropy on a slab guide [8], [9] and mode coupling in general anisotropic guides [10] have been studied recently, but no reported result for rectangular guides is known to the present author. The difficulty in obtaining an exact analytical solution for the propagation constant for a simpler isotropic rectangular guide lies in matching boundary conditions everywhere. A lengthy numerical method [11] may be used, but an approximate solution [12], [13] offers better insight to the propagation and field behavior of the modes.

Manuscript received May 30, 1979; revised October 3, 1979.
The author is with the Electronics and Communication Sciences Unit, Indian Statistical Institute, Calcutta 700, 035 India.

0018-9480/80/0300-0231$00.75 ©1980 IEEE

The natural modes of rectangular dielectric guides are all hybrid, with three components of both the electric and magnetic fields present. For weakly guiding structures they can be classified in EH^x_{nm}(x-polarized) and EH^y_{nm}(y-polarized) modes. For anisotropic guides, the TE and TM parts of hybrid mode fields have different transverse wave numbers and a single transcendental expression for the propagation constant cannot be derived by matching the approximate fields along the boundary. A lengthy numerical solution is possible also in this case, but the following approximate analysis can be useful in finding the propagation behavior of the guides.

II. Theory

The sectional diagram of the guide of size $a \times b$ is shown in Fig. 1. Consider at present that the guide is straight. The refractive indexes inside and outside the outline are $[n]$ and n_2, respectively. $[n]$ is a diagonal tensor with entries n_x, n_y, and n_z in x, y, and z direction, respectively. For uniaxial case $n_x = n_y$ and propagation is possible if min $n_i > n_2, i = x,y$. If $|n_i/n_2 - 1| \ll 1$, it can be shown that continuity of fields, except at the region shown hatched in Fig. 1, gives the propagation constant for straight isotropic $[n_x = n_y = n_z]$ waveguide to sufficient accuracy [12]. If this propagation constant is known and if the anisotropic guide is assumed to be a perturbation of the isotropy, then the change in propagation constant of the anisotropic guide from that of the isotropic one can be derived by a coupled mode theory using the ideal mode approach given below.

The electric and magnetic fields of the anisotropic guide satisfy the following Maxwell's equations for a time factor exp $[iwt]$

$$\overline{H} = iw\overline{D}, \qquad \overline{D} = \epsilon_0[n^2]\overline{E} \tag{1}$$

$$\overline{E} = -iw\mu_0\overline{H}. \tag{2}$$

Decomposing them into transverse components gives

$$-(1/iw\mu_0)\nabla_{t\wedge}(\nabla_{t\wedge}\overline{E}_t) + (\hat{e}_{z\wedge}\partial\overline{H}_t/\partial z) = iw\overline{D}_t \tag{3}$$

$$(1/iw\epsilon_0)\nabla_{t\wedge}\left[(1/n_z^2)\nabla_{t\wedge}\overline{H}_t\right] + (\hat{e}_{z\wedge}\partial\overline{E}_t/\partial z) = -iw\mu_0\overline{H}_t \tag{4}$$

where subscript t denotes transverse component, $\overline{E}, \overline{H}, \mu_0, \epsilon_0$ denote the electric and magnetic field, free space permeability, and permittivity, respectively, and $\hat{e}_z$ represents a unit vector in z direction.

For an isotropic guide we get, using $n_x = n_y = n_z = n_0$,

$$-(1/iw\mu_0)\nabla_{t\wedge}(\nabla_{t\wedge}\overline{E}_{0t}) - i\beta_0(\hat{e}_{z\wedge}\overline{H}_{0t}) = iw\overline{D}_{0t} \tag{5}$$

$$(1/iw\epsilon_0)\nabla_{t\wedge}\left[(1/n_0^2)\nabla_{t\wedge}\overline{H}_{0t}\right] - i\beta_0(\hat{e}_{z\wedge}\overline{E}_{0t}) = -iw\mu_0\overline{H}_{0t} \tag{6}$$

where β_0 is the propagation constant of isotropic guide. The subscript 0 has been used to distinguish the isotropic fields.

In coupled mode theory [14] the perturbed fields are represented as a sum of an infinite number of guided and radiation mode fields of the ideal guide. The representation gives the amount of power coupled among different guided and radiation modes. Since the perturbation in the present case is directional and independent of position in the guide, it does not produce coupling between different ideal modes, but changes the propagation constant of each ideal mode of the isotropic guide. So, let

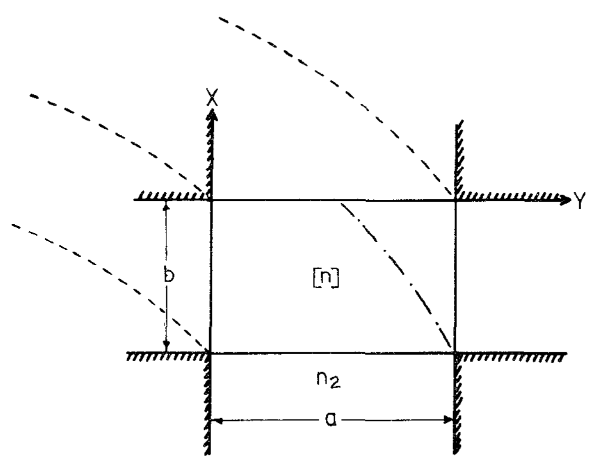

Fig. 1. Anisotropic dielectric rectangular guide structure.

$$\overline{E}_t = a_0(z)\overline{E}_{0t} \tag{7}$$

$$\overline{H}_t = b_0(z)\overline{H}_{0t}. \tag{8}$$

The assumptions (7) and (8) remains valid to the first order if the change in propagation constant stated above is small, i.e., if anisotropy does not change the transverse nature of the fields of the isotropic guide appreciably. This is so for the weakly anisotropic case $[1 - n_x/n_z] \ll 1$.

Using (5) and (6) and (7) and (8), (3) and (4) become

$$(db_0/dz + i\beta_0 a_0)(\hat{e}_{z\wedge}\overline{H}_{0t}) - iw(a_0\overline{D}_{0t} - \overline{D}_t) = 0 \tag{9}$$

$$(da_0/dz - i\beta_0 b_0)(\hat{e}_{z\wedge}\overline{E}_{0t}) + \frac{1}{iw\epsilon_0} b_0\nabla_{t\wedge}\left[n_z^{-2} - n_0^{-2}\right]\left[\nabla_{t\wedge}\overline{H}_{0t}\right] = 0. \tag{10}$$

Incorporating the power orthogonality relations in (9) and (10) we get

$$db_0/dz + i\beta_0 a_0 = k a_0 \tag{11}$$

$$da_0/dz + i\beta_0 b_0 = k b_0 \tag{12}$$

where

$$k = \frac{\epsilon_0 w}{iP}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[(n_x^2 - n_0^2)E_{0x}E_{0x}^* + (n_y^2 - n_0^2)E_{0y}E_{0y}^*\right]dxdy \tag{13}$$

$$k = \frac{-1}{iw\epsilon_0 P}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{H}_{0t}^*\nabla_{t\wedge} \cdot\left[\left(\frac{1}{n_z^2} - \frac{1}{n_0^2}\right)\nabla_{t\wedge}\overline{H}_{0t}\right]dxdy \tag{14}$$

with P as the power flowing in z direction of the isotropic guide, i.e.,

$$P = \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e_z\cdot\overline{E}_{0t\wedge}\overline{H}_{0t}^* dxdy. \tag{15}$$

After a lengthy reordering as in [14], equations (11)–(14)

can be written as

$$da_0/dz = -i\beta_0 a_0 + K_t a_0 \tag{16}$$

where

$$K_t = \frac{w\epsilon_0}{iP}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[(n_x^2-n_0^2)E_{0x}E_{0x}^* + (n_y^2-n_0^2)E_{0y}E_{0y}^* + \left(\frac{n_0^2}{n_z^2}\right)(n_z^2-n_0^2)E_{0z}E_{0z}^*\right]dxdy. \tag{17}$$

Let $da_0/dz = -i\beta a_0$, where $\beta = \beta_0 + \Delta\beta$ is the propagation constant of the anisotropic guide. K_t vanishes outside the guide where $n_i = n_0, i = x,y,z$. Then (16) becomes

$$\Delta\beta = \frac{w\epsilon_0}{P}\int_{-b}^{0}\int_{0}^{a}\left[(n_x^2-n_0^2)E_{0x}E_{0x}^* + (n_y^2-n_0^2)E_{0y}E_{0y}^* + \left(\frac{n_0^2}{n_z^2}\right)(n_z^2-n_0^2)E_{0z}E_{0z}^*\right]dxdy. \tag{18}$$

The equation is quite general to account for uniaxial as well as biaxial anisotropic guides and gives an interesting condition of $\Delta\beta=0$ for biaxial case. If the ideal guide is of square cross section, $E_{0x}E_{0x}^* = E_{0y}E_{0y}^*$ is satisfied for the modes with same wave number in x and y direction. Now, if we choose $n_0 = n_z$ and the biaxial guide is such that $n_z = [(n_x^2+n_y^2)/2]^{1/2}$, then $\Delta\beta = 0$.

For uniaxial guide, we choose $n_0 = n_x = n_y$. Then (18) reduces to

$$\Delta\beta = \frac{w\epsilon}{P}\int_{-b}^{0}\int_{0}^{a}\left(\frac{n_0^2}{n_z^2}\right)(n_z^2-n_0^2)E_{0z}E_{0z}^* dxdy. \tag{18a}$$

The same expression (18a) can be obtained using a variational theorem [16] if it is understood that the effective refractive index variation in the z direction due to anisotropy is $(n_0/n_z)(n_z^2-n_0^2)^{1/2}$ and this variation effects the change $\Delta\beta$ in the propagation constant of the isotropic guide.

The integral in (18a) can very easily be evaluated if the approximate expressions [12] for the isotropic rectangular guide fields are used. The fields are

$$E_{0z} = A\cos k_x(x+\xi)\cos k_y(y+\eta) \tag{19}$$

$$H_{0z} = -A(\epsilon_0/\mu_0)^{1/2} n_0^2 (k_y/k_x)(k/\beta_0) \cdot \sin k_x(x+\xi)\sin K_y(y+\eta) \tag{20}$$

where $K = 2\pi/\lambda$. K_x and k_y are wave numbers with ξ and η as the phase factors in x and y directions, respectively. The phase factors account for the phase of the mode at the center of the guide. For HE_{mn}^x modes k_x and k_y are given, respectively, by

$$k_x = \frac{m\pi}{b}\left[1+\frac{n_2^2\lambda}{n_0^2\pi b(n_0^2-n_2^2)^{1/2}}\right]^{-1} \tag{21}$$

$$k_y = \frac{n\pi}{a}\left[1+\frac{\lambda}{\pi a(n_0^2-n_2^2)^{1/2}}\right]^{-1} \tag{22}$$

and β_0 is found from $k_x^2 + k_y^2 = K^2 n_0^2 - \beta_0^2$.

III. Results and Discussion

The change in propagation constant of HE_{11}^x mode of uniaxial guide is shown in Figs. 2 and 3. In Fig. 2, $\Delta\beta/\beta_0$ is plotted against a/λ for different n_0 keeping n_0/n_2 and n_0/n_z constant, where β_0 and λ are the free space propagation constant and wavelength, respectively. It is noted that the variation of propagation constant decreases exponentially with the dimension of the guide. This is because the fields become more transverse in nature and $E_{0z}E_{0z}^*/P$ decreases with increased dimension. For small a/λ, where the mode is near cutoff, the approximate relations (19)–(22) are not valid. A harmonic series representation [11] of E_{0z} and H_{0z} can be used in (18a) to find the change in propagation constant. In Fig. 3, $\Delta\beta/\beta_0$ is plotted against a/λ for different n_z keeping n_0 and n_2 constant. It is seen that the variation in propagation constant is larger with larger n_z. This shows the effect of anisotropy in changing the propagation behavior of isotropic guide. The result for the first higher order mode HE_{12}^x we also calculated, but $\Delta\beta/\beta_0$ does not differ more than 0.1 percent from that of HE_{11}^x mode.

An examination of equation (18a) shows that $\Delta\beta/\beta_0$ may be positive or negative depending on whether n_z is larger than n_0 or not. The result may be useful in matching the propagation constant of one guide with that of another.

The theory can be extended to curved guides to the extent that guide mode orthogonality is still valid and normalization with respect to power P still holds. In that case Marcatill's approach [15] can be used to find the mode conversion and loss factor of the structure. For a curvature in y direction, i.e., axis of curvature parallel to the x axis, as shown in Fig. 1 by dotted lines, the transverse wave number k_x remains unchanged while k_y is modified to

$$k_y' = k_y(1+C-jI) \tag{23}$$

where k_x and k_y are given by (21) and (22) and

$$I = \frac{\beta}{2k_y^2 R}\left(1-\frac{n_2^2}{n_0^2}\right)^{-1/2}\left(\frac{n_2 k_y b}{n_0}\right)^2\left(\frac{A}{\pi b}\right)^3\left[1-\left(\frac{k_y A}{\pi}\right)^2\right]^{1/2} \cdot \frac{R'\exp -\frac{R'}{3}\left[1-\left(\frac{k_y A}{\pi}\right)^2\left(1+\frac{2c}{bk_y}\right)^2\right]}{1-\left[1-\left(\frac{n_2}{n_0}\right)^4\right]\left[\frac{k_y A}{\pi}\right]^2 + 2\left(\frac{n_2}{n_0}\right)^2\left(\frac{A}{b}\right)\left[1-\left(\frac{k_y A}{\pi}\right)^2\right]^{-1/2}} \tag{24}$$

$$C = \frac{1}{2k_y b}\left(\frac{\pi b}{A}\right)^3\frac{1}{R'} \tag{25}$$

where

$$R' = \frac{2\pi^3 R}{\beta^2 A^3}, \qquad A = \frac{\lambda}{2(n_0^2-n_2^2)^{1/2}}$$

and R is the radius of curvature.

To get an idea how the commonly known anisotropic materials behave, the result for straight and curved guide

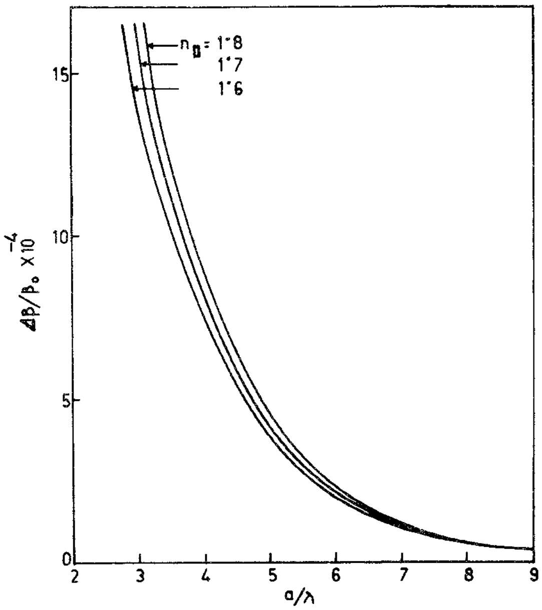

Fig. 2. Variation of $\Delta\beta/\beta_0$ of HE^x_{11} mode with a/λ for different n_0 with $n_2/n_0 = 0.9375$, $n_z/n_0 = 1.0125$, and $a = b$.

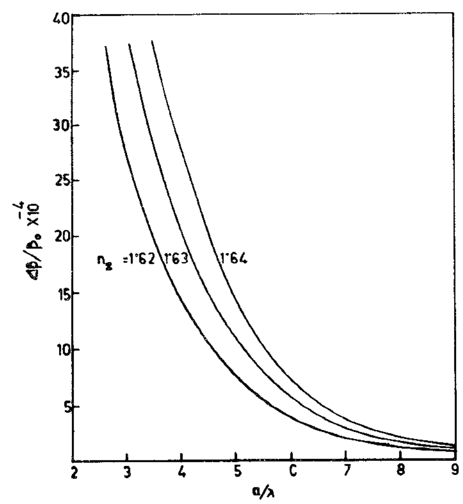

Fig. 3. Variation $\Delta\beta/\beta$ of HE^x_{11} mode with a/λ for different n_z with $n_0 = 1.6$, $n_2 = 1.5$, and $a = b$.

of dimensions $a = 5\mu$, $b = 2.5\mu$ at $\lambda = 0.6328\mu$, $n_2 = 0.95n_0$, and $R = 1000\lambda$ is given in Table I. It is seen that the change in propagation constant from the isotropic to anisotropic case increases with the anisotropy factor $A = n_2/n_0$. The loss function I is more dependent on the transverse refractive index n_0 and increases with n_0 decreasing. However, the variation of I with that of A seems to be less prominent.

Equation (18) can also be applied to the slab and circular cylindrical anisotropic guides. For cylindrical guides, all parameters in (18) has to be transformed into cylindrical coordinate system. Furthermore, since exact analytical relation is available for the propagation constant of a uniaxial cylindrical guides [3], equation (18) is applicable to the biaxial case with better accuracy. The analysis, however, is restricted to materials with diagonal refractive index tensor. For a more general case, i.e., for the nondiagonal entries in refractive index tensor, the modes are coupled among themselves as they propagate and the simple assumptions (7) and (8) no longer remain valid.

TABLE I
PROPAGATION CONSTANT CHANGE AND LOSS FUNCTION OF SOME ANISOTROPIC MATERIAL GUIDES FOR HE^x_{11} MODE

Material	n_0	A	$\Delta\beta/\beta_0$ $\times 10^{+3}$	I $\times 10^{+2}$
Titania [TiO_2]	2.71	0.8941	7.8926	2.2612
Calcite [$CaCO_3$]	1.66	0.8975	3.7920	3.5851
Tourmaline	1.64	0.9879	0.3800	3.6218

ACKNOWLEDGMENT

The author wishes to thank Prof. D. Dutta Majumder of ECS Unit, Indian Statistical Institute, Calcutta and Dr. D. K. Paul of Gordon Mckay Laboratory, Harvard University, Cambridge, MA for their interest, and J. Gupta for typing the manuscript.

REFERENCES

[1] P. R. Longakar and C. S. Roberts, "Propagation constants for TE and TM surface waves on an anisotropic dielectric cylinder," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-11, pp. 543–546, Nov. 1963.
[2] D. K. Paul, "On power flow through anisotropic circular cylindrical dielectric rod waveguide," *JITE*, vol. 13, no. 11, pp. 437–451, Nov. 1967.
[3] F. J. Rosenbaum, "Hybrid modes on anisotropic dielectric rods," *IEEE J. Quantum Electron.* vol. QE-1, pp. 367–374, Dec. 1965.
[4] R. K. Shevgaonkar, "Guided wave optics in anisotropic dielectric," M. Tech. thesis, Indian Inst. Technol., Kanpur, India, 1977.
[5] J. Rosenbaum and L. Kraus, "Propagation in a weakly anisotropic wave guide," *Appl. Opt.*, vol. 16, no. 8, pp. 2204–2211, Aug. 1977.
[6] J. R. Cozens, "Propagation in cylindrical fibers with anisotropic crystal cores," *Electron. Lett.*, vol. 12, no. 10, pp. 413–415, Aug. 1976.
[7] B. B. Chaudhuri and D. K. Paul, "Propagation through hollow cylindrical anisotropic dielectric guides," *IEEE Trans. Microwave Theory Tech.*, vol. 27, pp. 170–172, Feb. 1979.
[8] W. Burns and T. Warner, "Mode dispersion in uniaxial optical waveguides," *J. Opt. Soc. Amer.*, vol. 64, no. 4, pp. 441–446, 1974.
[9] S. Yamamoto, Y. Koyamada, and T. Makimoto, "Normal mode analysis of anisotropic and gyrotropic thin film waveguides for integrated optics," *Appl. Opt.*, vol. 41, no. 12, pp. 5091–5097, Dec. 1972.
[10] D. Marcuse, "Coupled-mode theory for anisotropic optical waveguides," *Bell. Syst. Tech. J.*, vol. 54, no. 6, pp. 985–995, 1975.
[11] J. E. Goell, "A circular harmonic computer analysis of rectangular dielectric waveguides," *Bell. Syst. Tech. J.* vol. 48, no. 7, pp. 2133–2160, Sept. 1969.
[12] E. A. J. Marcatill, "Dielectric rectangular wave guides and directional couplers for integrated optics," *Bell. Syst. Tech. J.*, vol. 48, pp. 2071–2102, Sept. 1969.
[13] B. B. Chaudhuri and D. K. Paul, "Wave propagation through a hollow rectangular anisotropic dielectric guide," *IEEE J. Quantum Electron.*, vol. QE-14, pp. 557–560, Aug. 1968.
[14] D. Marcuse, *Theory of Dielectric Optical Waveguides*. pp. 95–111, New York: Academic, 1974.
[15] E. A. J. Marcatill, "Bends in optical dielectric guides," *Bell. Syst. Tech. J.*, vol. 48, no. 7, pp. 2103–2132, Sept. 1968.
[16] W. P. Alis, S. J. Buchsbaum, and A. Bers, *Wages in Anisotropic Plasma.* New York: Wiley, 1962.